NUMERATION SYSTEMS WITH FINITE TYPE CONDITION

(Part II: Demonstration)

Demonstration

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$\Rightarrow \xi_n \in Y$.

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Proof of item(a)

$$\left[\frac{t_1}{q} + \cdots + \frac{t_n}{q^n} + \sum_{n+1}^{\infty} \frac{t_i}{q^i} = y\right] \Rightarrow \left[q^n \sum_{n+1}^{\infty} \frac{t_i}{q^i} \overset{\star}{=} \left(yq^n - t_1 q^{n-1} - \cdots - t_n\right) =: \xi_n\right].$$

L.H.S.: $\left| q^n \sum_{n+1}^{\infty} \frac{t_i}{q^i} \right| \leq \sum_{1}^{\infty} \frac{\text{const.}}{q^i} =: C_0$.

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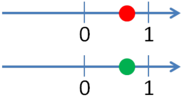
Personal identity as an analogy



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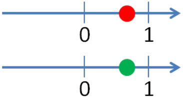
To argue that two people are the same person...



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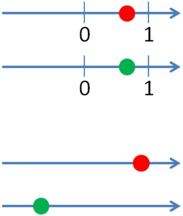

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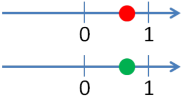

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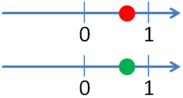

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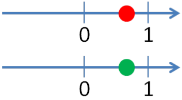

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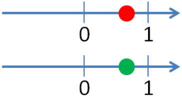

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Personal identity as an analogy

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Criteria: Same passport no.; Same DNA; etc.
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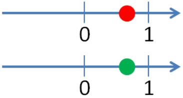

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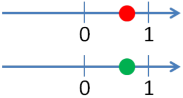

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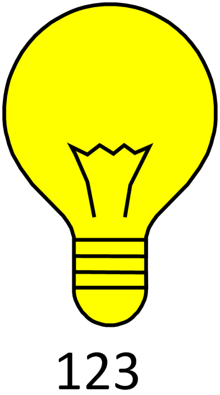

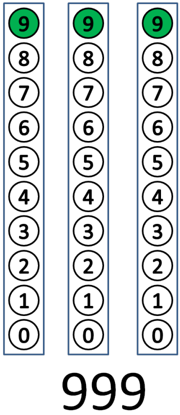

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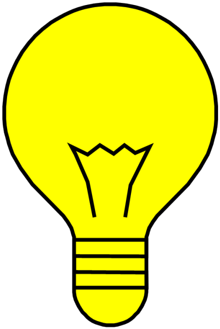

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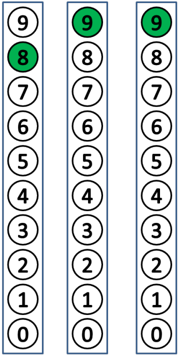

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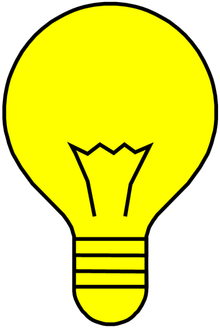

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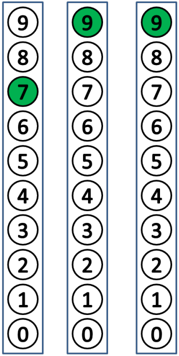

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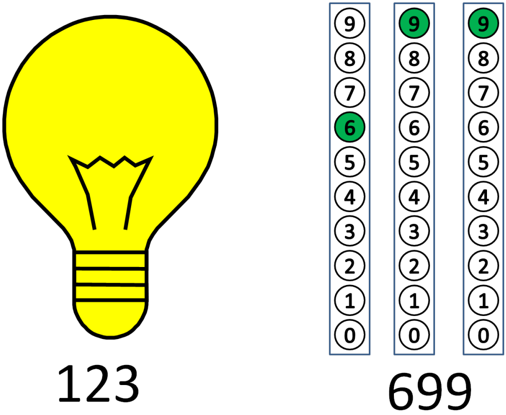

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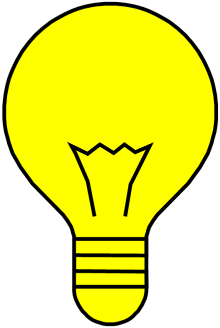

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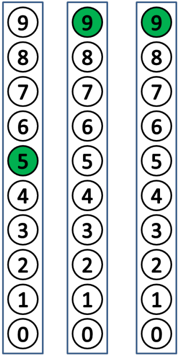

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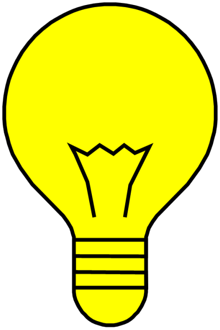

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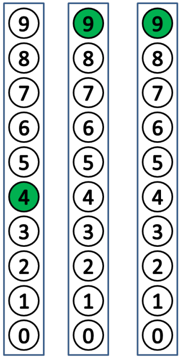

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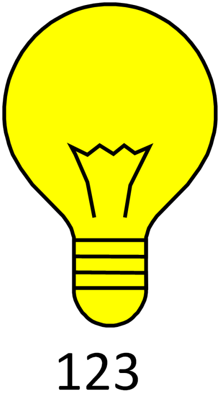

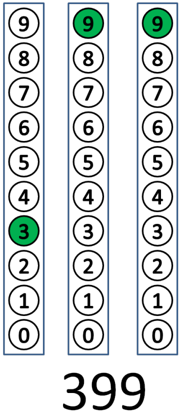

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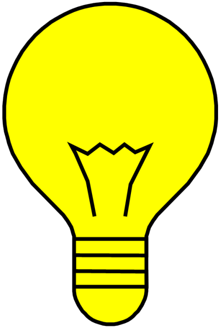

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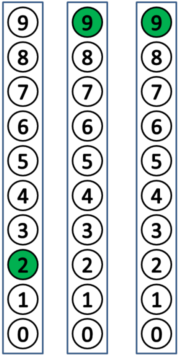

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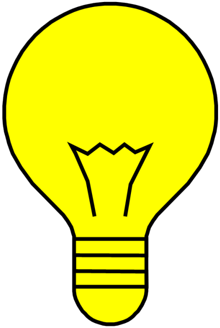

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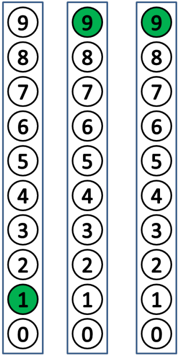

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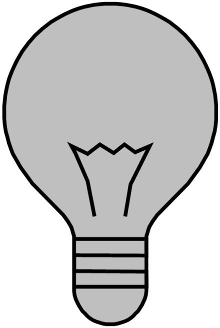

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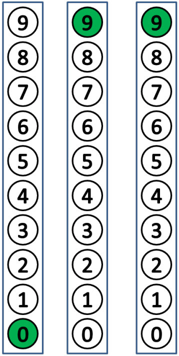

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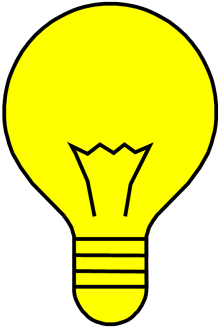

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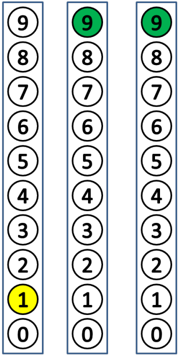

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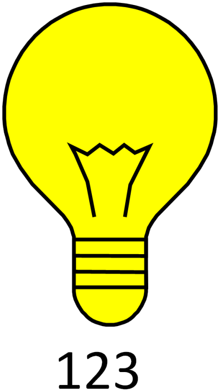

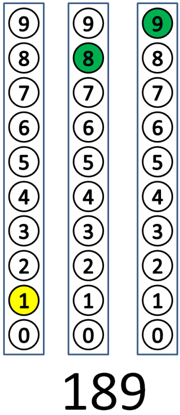

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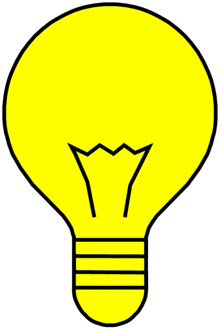

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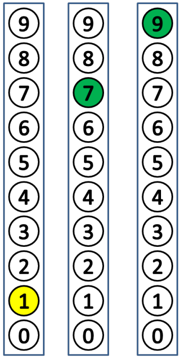

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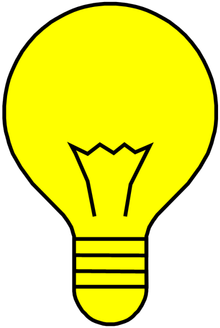

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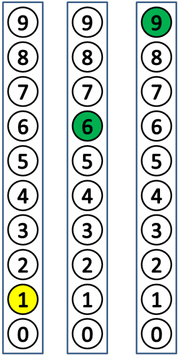

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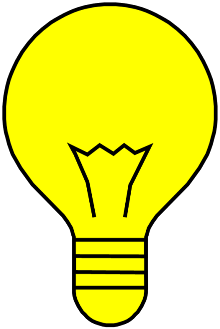

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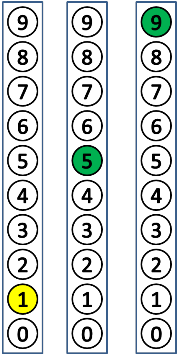

159

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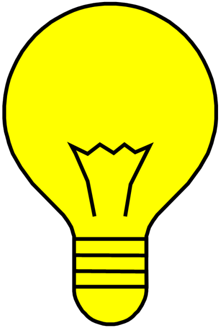

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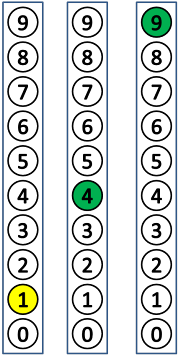

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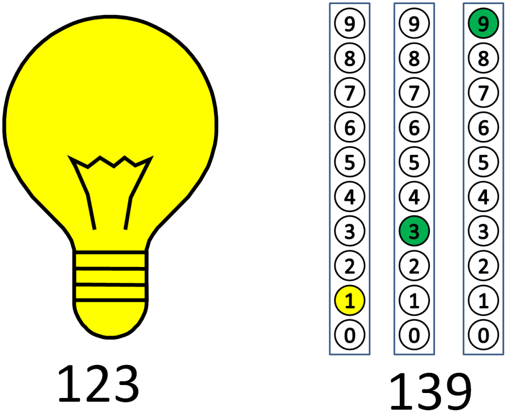

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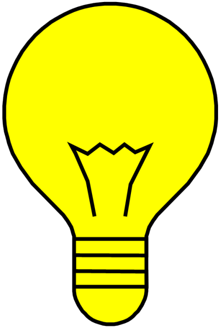

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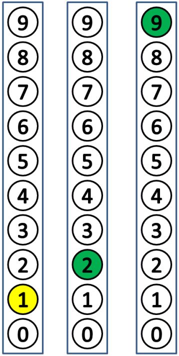

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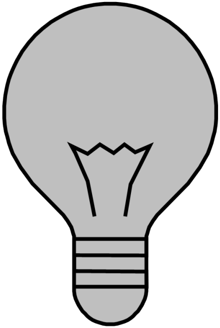

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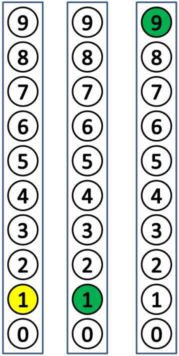

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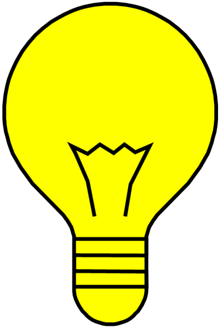

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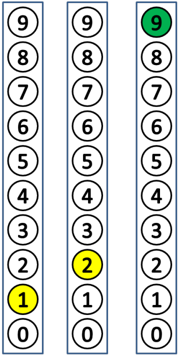

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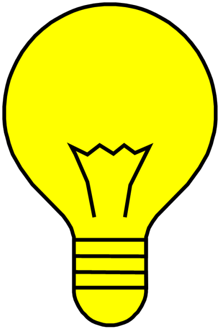

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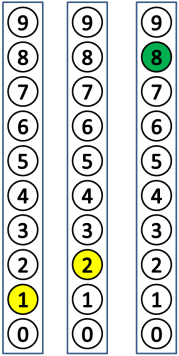

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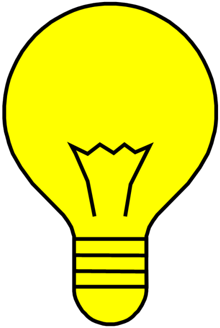

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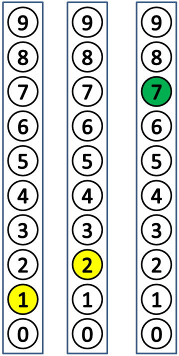

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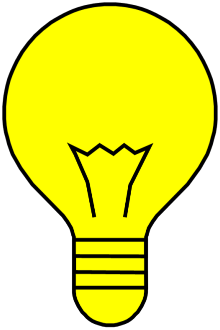

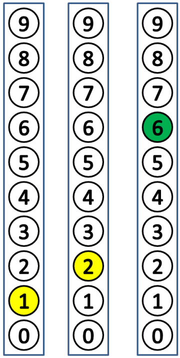

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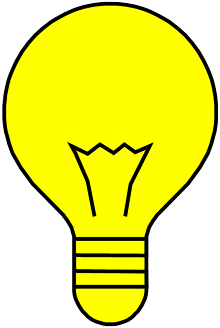

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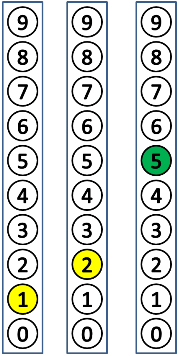

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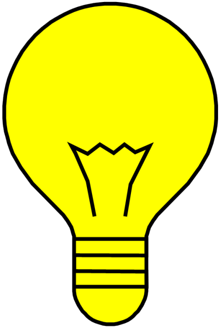

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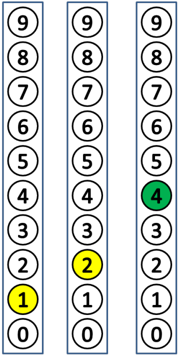

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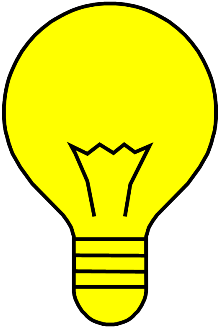

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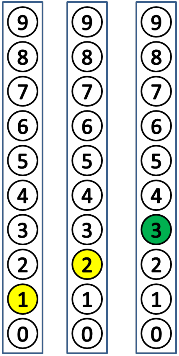

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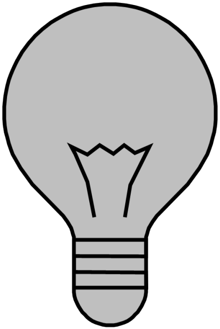

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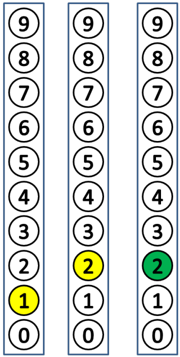

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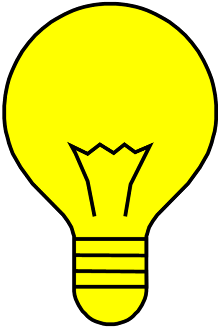

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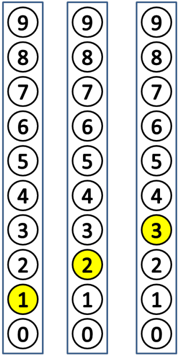

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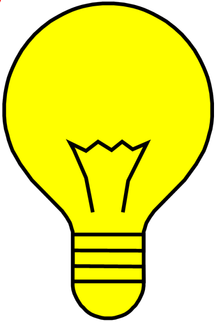
Another switch box

Again we want supply = demand

(R.H.S. = L.H.S.)



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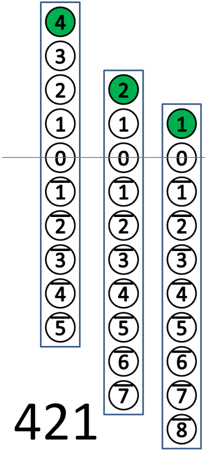

421

($\bar{x}$ = -x)

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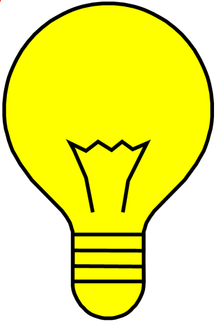

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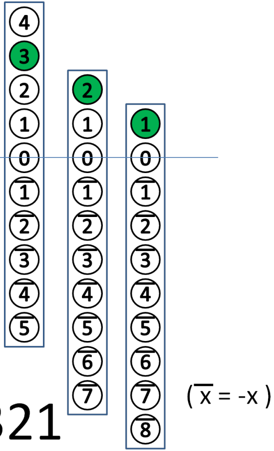

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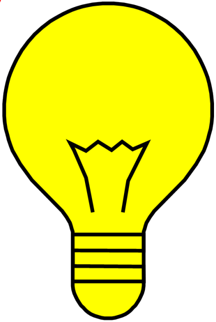

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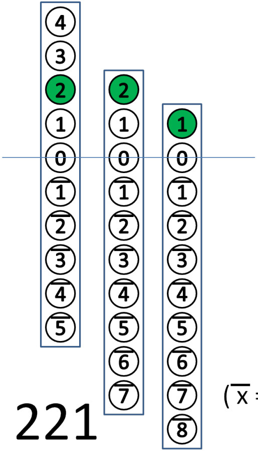

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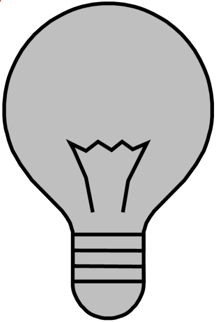

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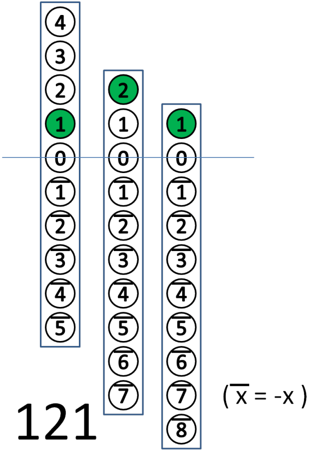

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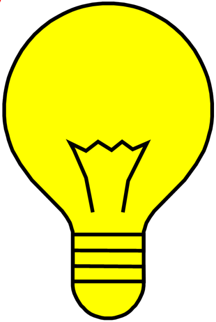
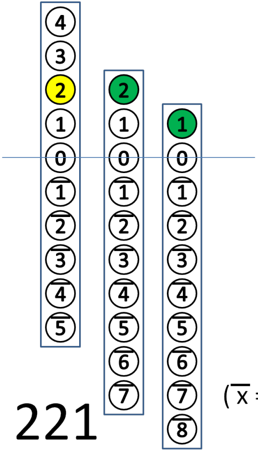


123



221

$(\overline{x} = -x)$

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

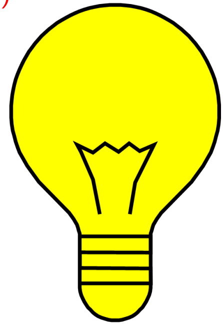
It gives a two-step strategy of showing $|A| \leq 1$:

(i) Find an expansion $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$ (ii) Show that $\sigma_A(y) \neq \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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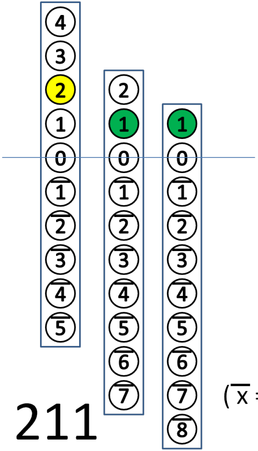

211

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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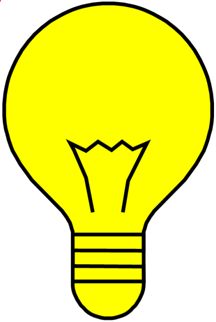
Another switch box

Again we want supply = demand

(R.H.S. = L.H.S.)



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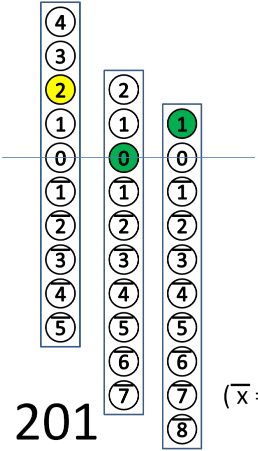

201

($\overline{x}$ = -x)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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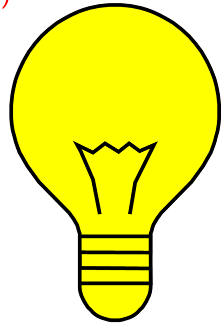
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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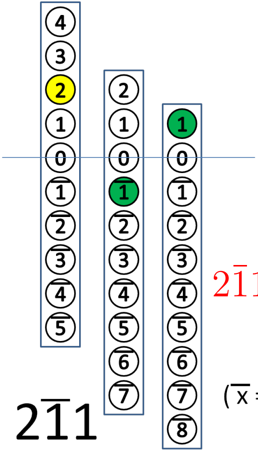

$2\bar{1}1$

$$2\bar{1}1 = 201 - 10$$
$$= 191$$

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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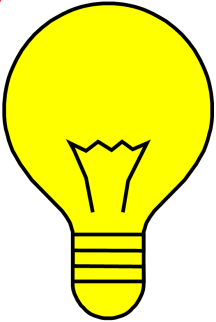
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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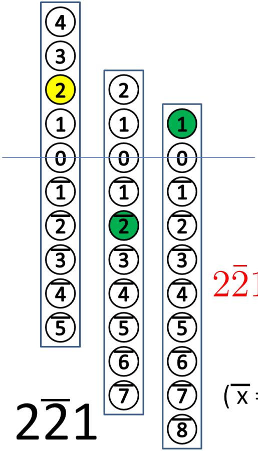

$2\bar{2}1$

$$2\bar{2}1 = 201 - 20$$
$$= 181$$

($\bar{x}$ = -x)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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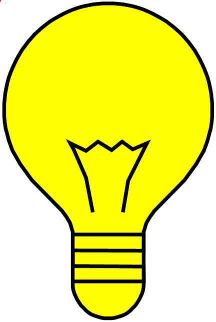
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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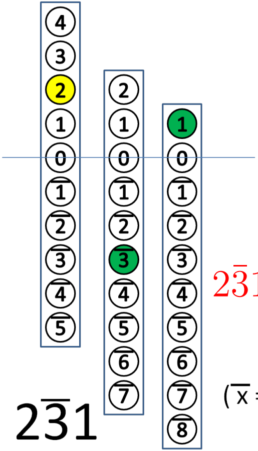

$2\bar{3}1$

$$2\bar{3}1 = 201 - 30$$
$$= 171$$

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

It gives a two-step strategy of showing $|A| \leq 1$:

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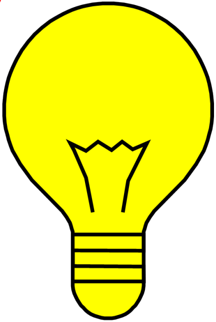
Another switch box

Again we want supply = demand

(R.H.S. = L.H.S.)



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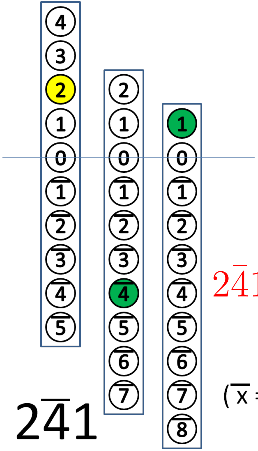

$2\bar{4}1$

$$2\bar{4}1 = 201 - 40$$
$$= 161$$

($\bar{x}$ = -x)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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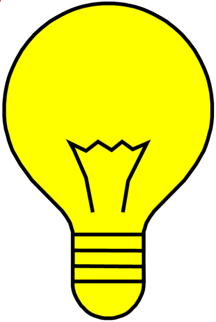
Another switch box

Again we want supply $=$ demand
(R.H.S. $=$ L.H.S.)



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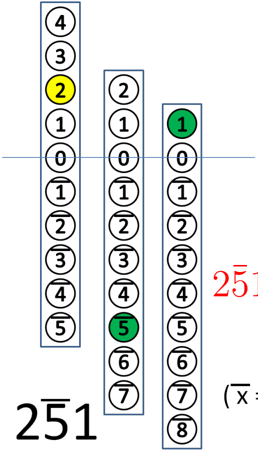

$2\bar{5}1$

$$2\bar{5}1 = 201 - 50$$
$$= 151$$

$(\bar{x} = -x)$

Proposition 1 (Consequence of FTC)
Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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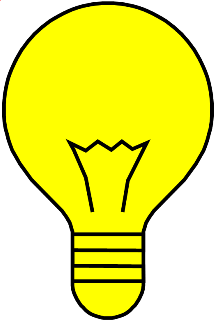
(i) Find an expansion $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$ (ii) Show that $\sigma_A(y) \neq \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

Another switch box

Again we want supply $=$ demand
(R.H.S. $=$ L.H.S.)



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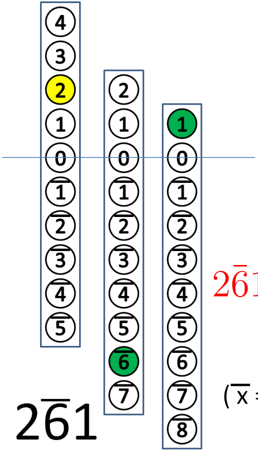

$2\bar{6}1$

$$2\bar{6}1 = 201 - 60$$
$$= 141$$

$(\bar{x} = -x)$

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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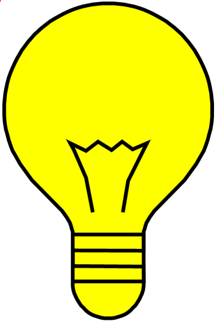
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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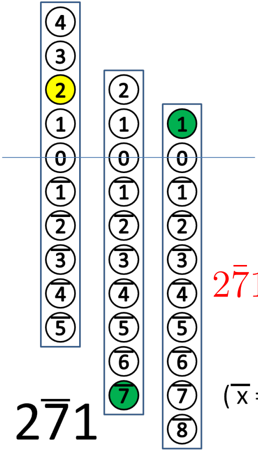


$2\bar{7}1$

$$2\bar{7}1 = 201 - 70 = 131$$

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

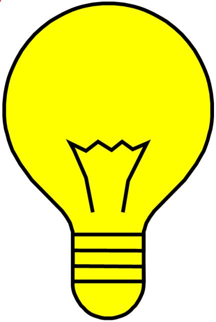
It gives a two-step strategy of showing $|A| \le 1$:

(i) Find an expansion $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$ (ii) Show that $\sigma_A(y) \neq \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

Another switch box

Again we want supply = demand

(R.H.S. = L.H.S.)



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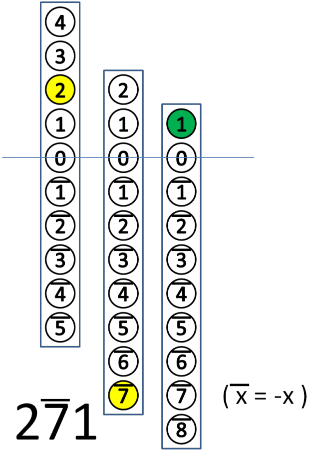

$2\overline{7}1$

Proposition 1 (Consequence of FTC)
Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

It gives a two-step strategy of showing $|A| \leq 1$:

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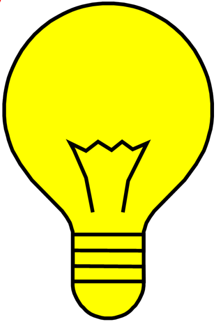
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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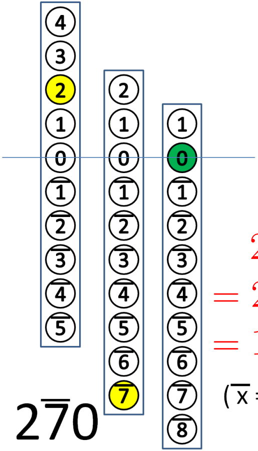

$2\bar{7}0$

$$2\bar{7}0$$
$$= 200 - 70$$
$$= 130$$

$(\bar{x} = -x)$

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in$ (D$-$D). If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

It gives a two-step strategy of showing $|A| \leq 1$:

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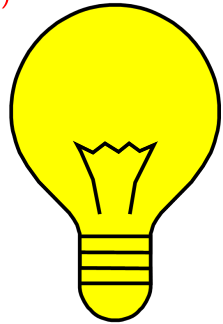
Another switch box

Again we want supply $=$ demand (R.H.S. $=$ L.H.S.)



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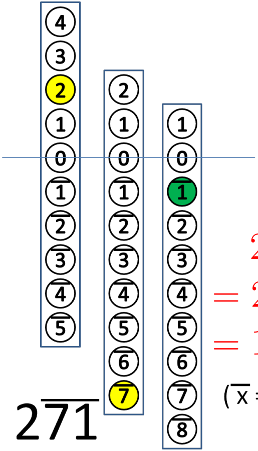

$2\overline{71}$

$$2\bar{7}\bar{1} = 200 - 70 - 1 = 129$$

$(\overline{x} = -x)$

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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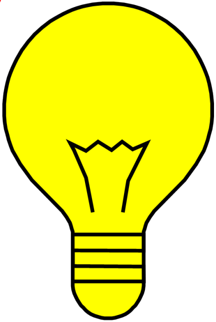
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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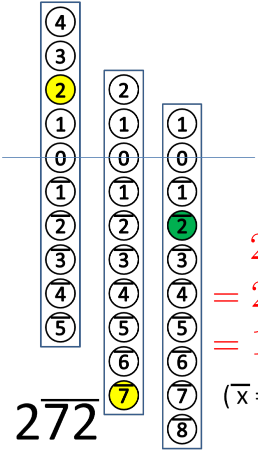

$2\bar{7}\bar{2}$

$$2\bar{7}\bar{2} = 200 - 70 - 2 = 128$$

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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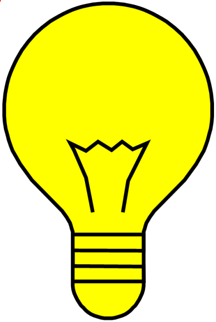
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



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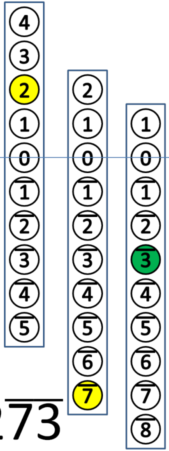

$2\overline{7}\overline{3}$

$$2\bar{7}\bar{3}$$
$$= 200 - 70 - 3$$
$$= 127$$

($\overline{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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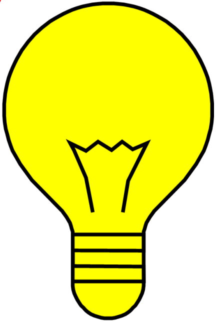
Another switch box

Again we want supply = demand

(R.H.S. = L.H.S.)



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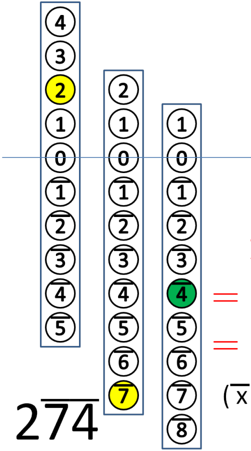

$2\overline{74}$

$$2\bar{7}\bar{4}$$
$$= 200 - 70 - 4$$
$$= 126$$

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

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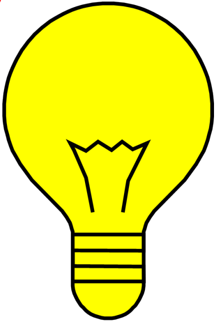
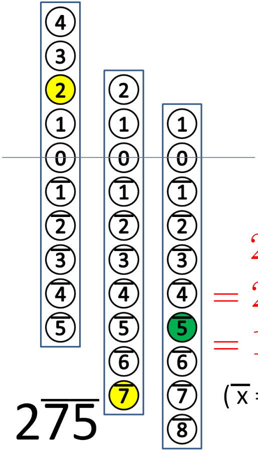
Another switch box

Again we want supply $=$ demand

(R.H.S. $=$ L.H.S.)



123



$2\bar{7}\bar{5}$

$$2\bar{7}\bar{5} = 200 - 70 - 5 = 125$$

($\bar{x} = -x$)

Proposition 1 (Consequence of FTC)

Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathsf{D}-\mathsf{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

It gives a two-step strategy of showing $|A| \leq 1$:

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Another switch box

Again we want supply $=$ demand

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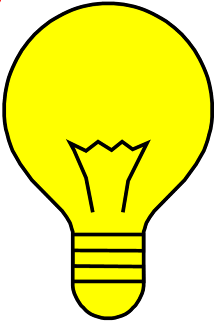
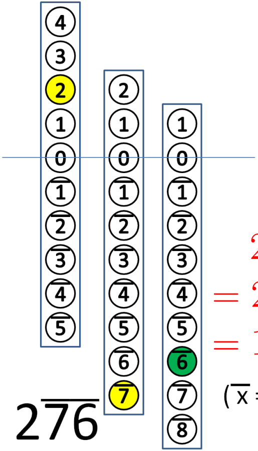


123



$2\bar{7}\bar{6}$

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$= 200 - 70 - 6$

$= 124$

$(\bar{x} = -x)$

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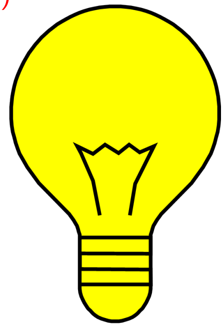

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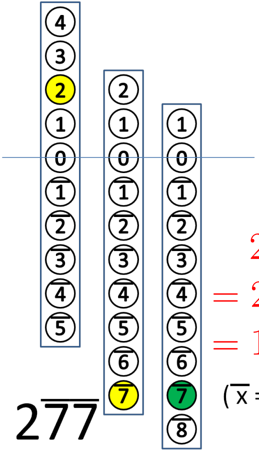

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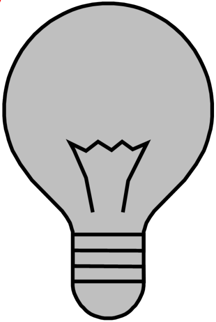

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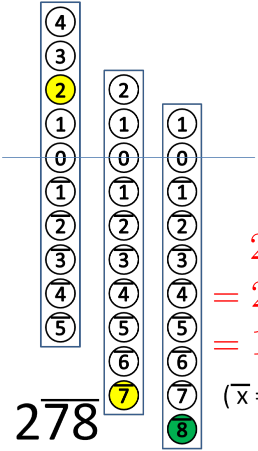

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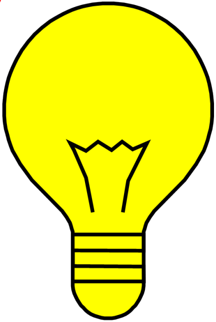

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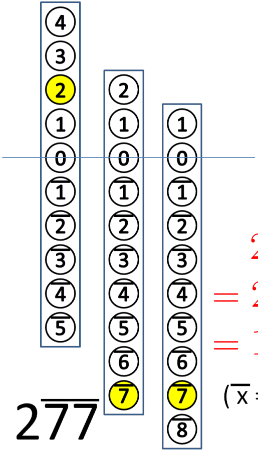

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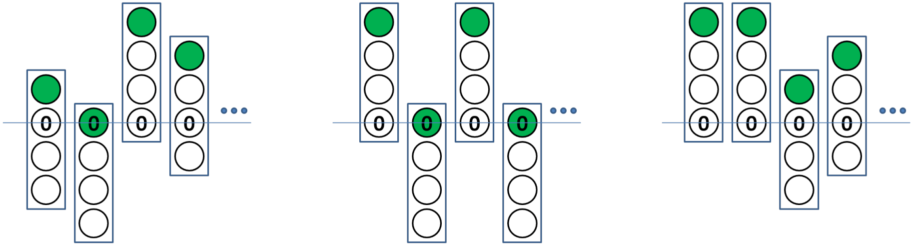
Lazy algorithm for step(i)

So we can consider various switch boxes...



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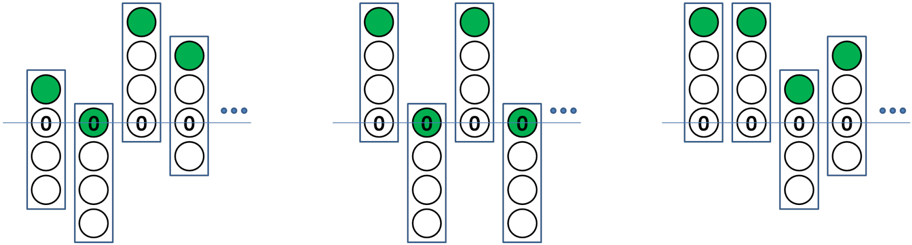

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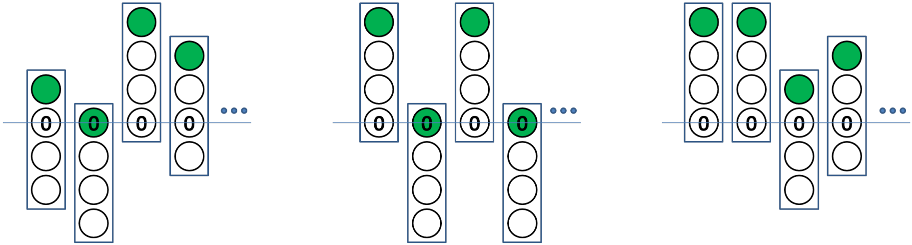

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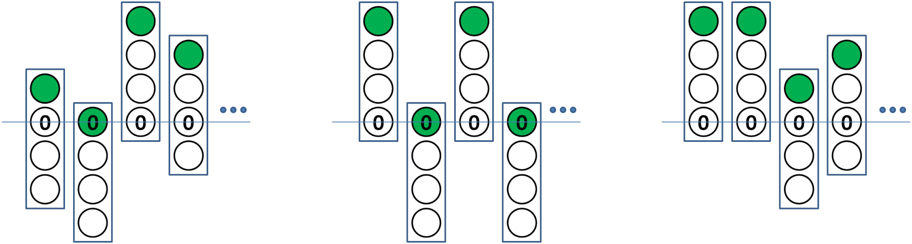

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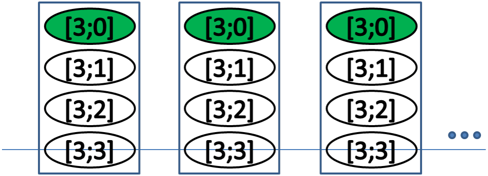

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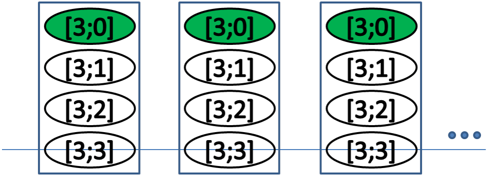

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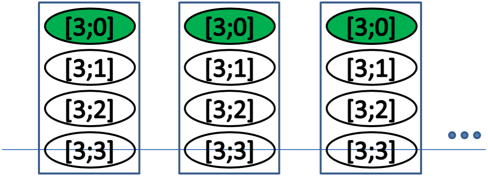

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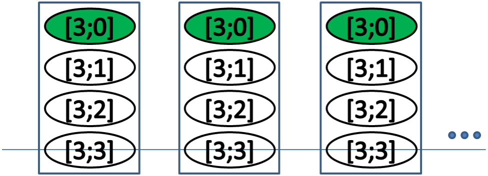

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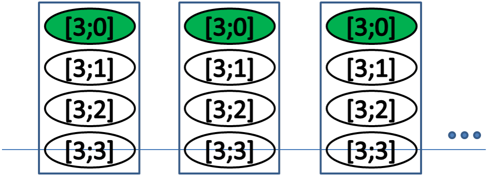

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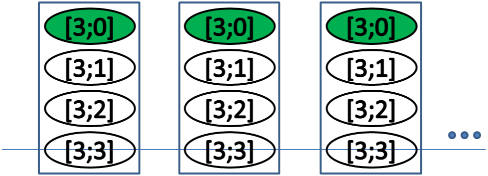

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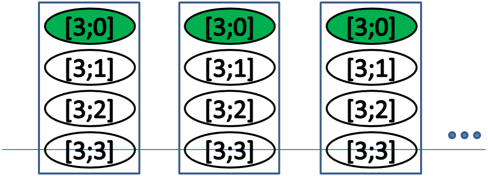

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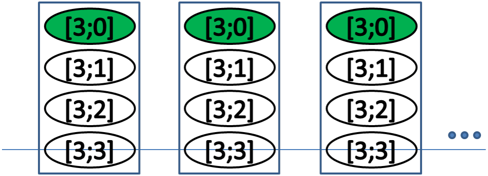

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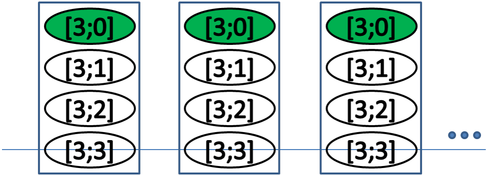

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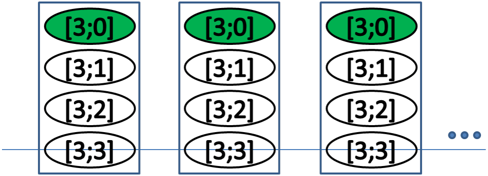

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Example $m = 5,\ q = 3 + \sqrt{3} \approx 4.732,$

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∂_0	∂_1	∂_2	∂_3	∂_4	∂_5
0	1	$q/3$	$2q/3-1$	$q-2$	$q-1$
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Let A be an algebraic conjugate of q, and $y \in Y$. Suppose Φ satisfies the FTC and $y = \sum_{i=1}^{\infty} \frac{t_i}{q^i}$, where $t_i \in (\mathrm{D}-\mathrm{D})$. If $|A| > 1$, then $\sigma_A(y) = \sum_{i=1}^{\infty} \frac{\sigma_A(t_i)}{A^i}$.

Proposition 2 (Special case $\langle 3012 \rangle$)

Let $A > 1$ be a real algebraic conjugate of q. Suppose Φ satisfies the FTC and $m = 3$. Then it is impossible that $\sigma_A(\partial_3) > \sigma_A(\partial_0) > \sigma_A(\partial_1) > \sigma_A(\partial_2)$.

So, "3012" is a forbidden pattern of $\{\sigma_A(\partial_i)\}_{i=0}^{m}$ if we want $A > 1$. The study of forbidden patterns and their avoidance allows us to give an example of Φ which satisfies the FTC and the associated q is NOT a PV number.

Example $m = 5$, $q = 3 + \sqrt{3} \approx 4.732$, and $\{\partial_i\}_{i=0}^{m}$ with $0 < \partial_{i+1} - \partial_i \leq 1$ are given by

∂_0	∂_1	∂_2	∂_3	∂_4	∂_5
0	1	$q/3$	$2q/3-1$	$q-2$	$q-1$
0	1	1.577	2.154	2.732	3.732

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It corresponds to the allowed pattern "125034": letting $A = 3 - \sqrt{3} \approx 1.267$, we have

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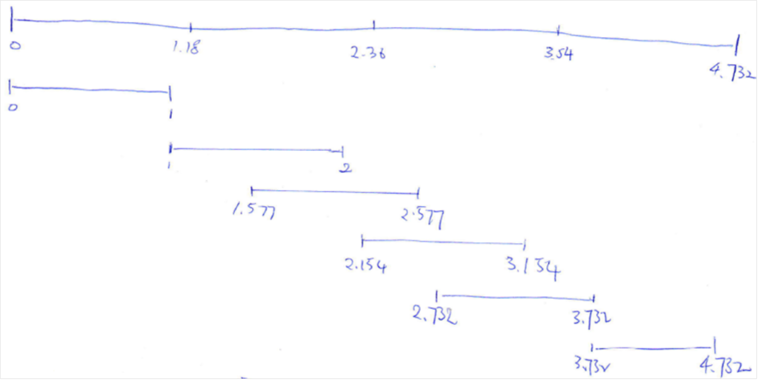
We end by visualizing it. When $\ell = 0$, $\left[q^{\ell}\mathsf{D} + \cdots + q\mathsf{D} + \mathsf{D}\right]$ gives the following.



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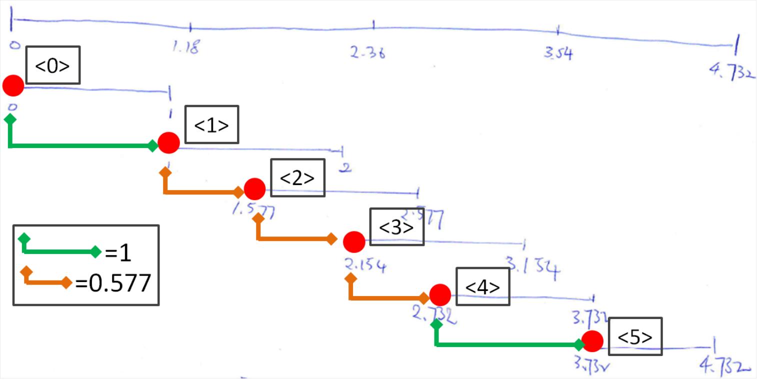

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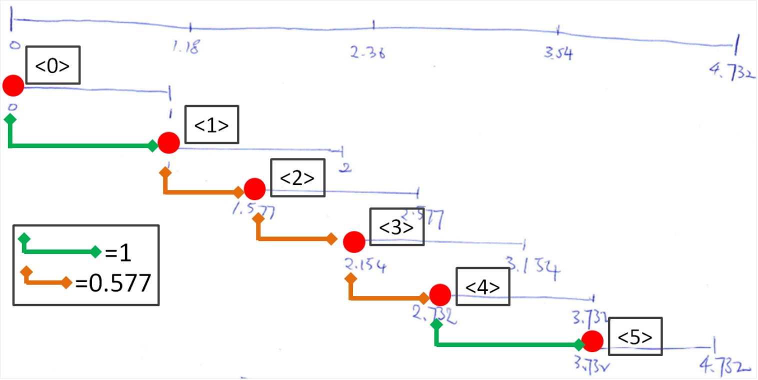

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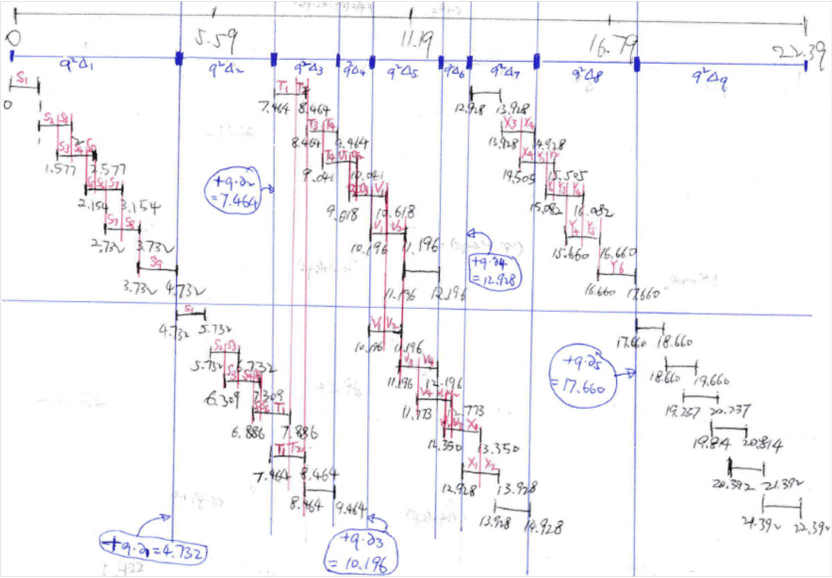

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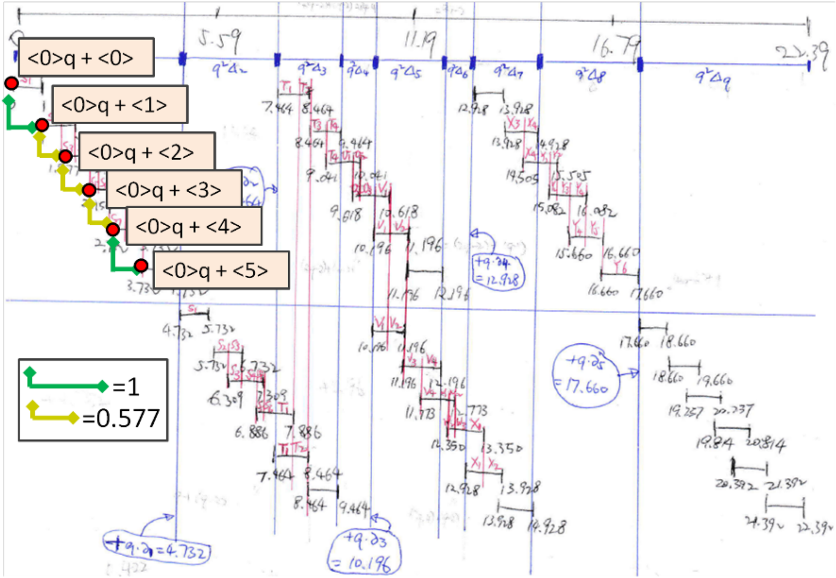

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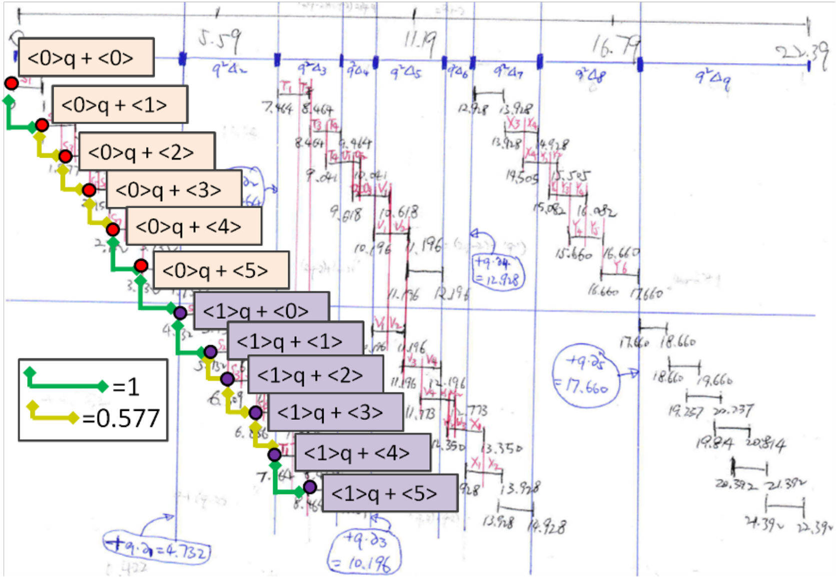

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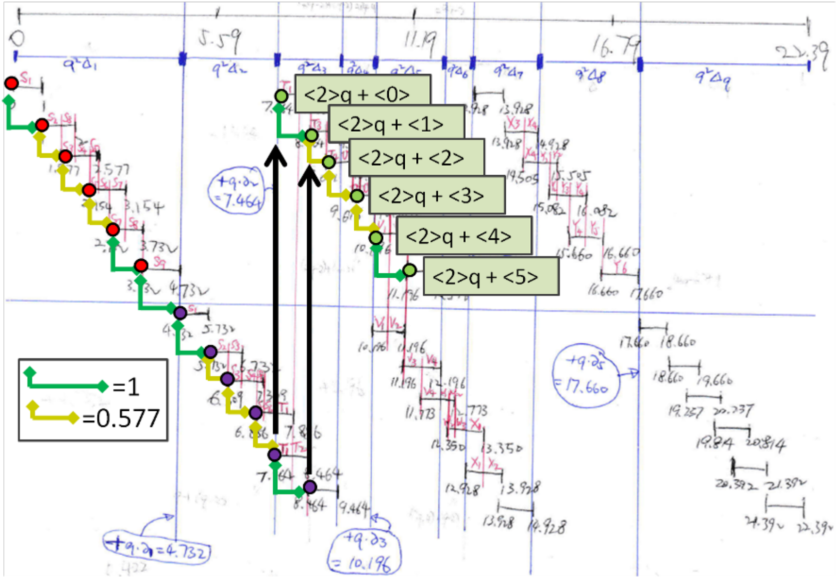

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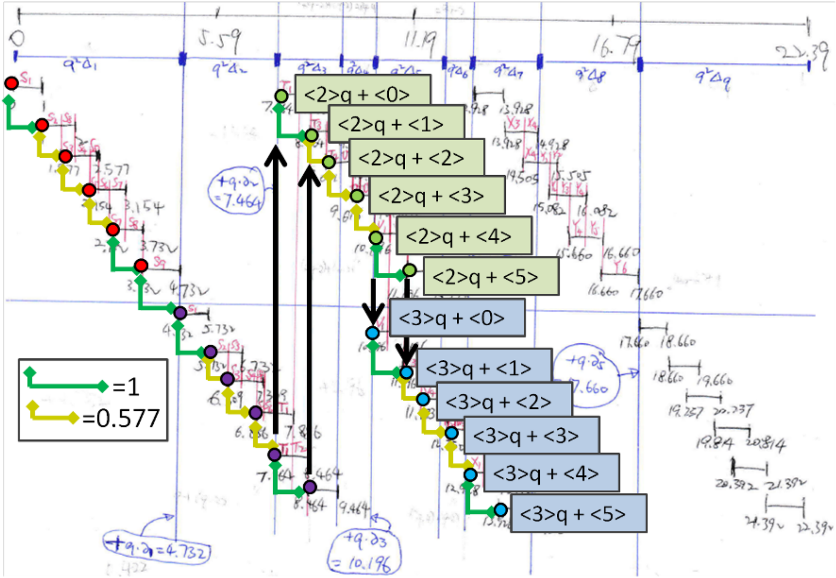

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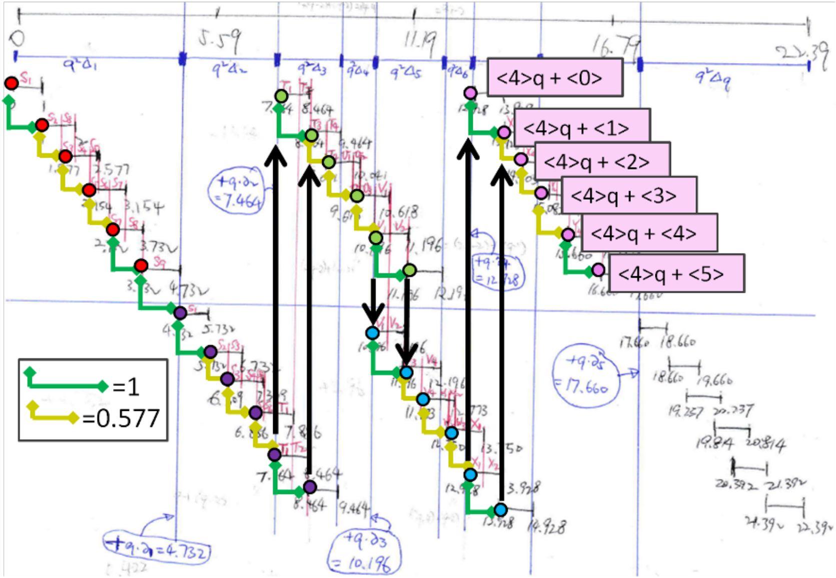

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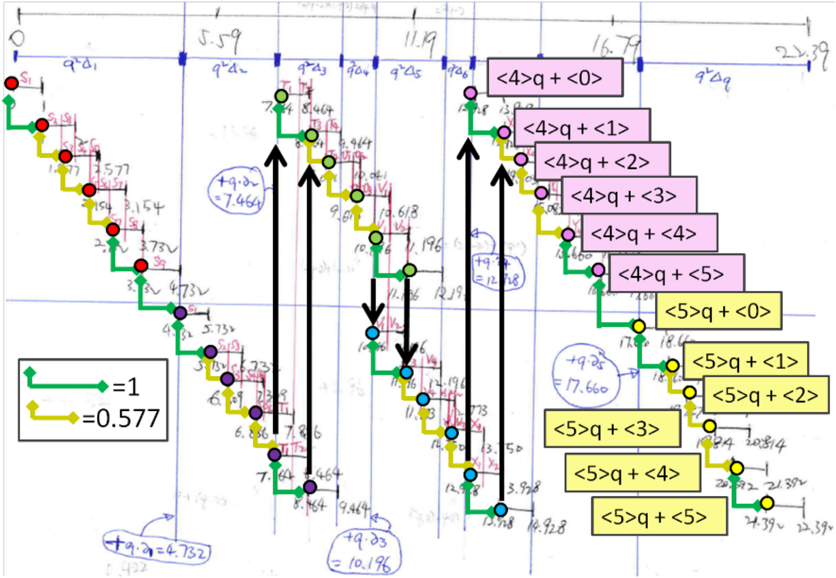

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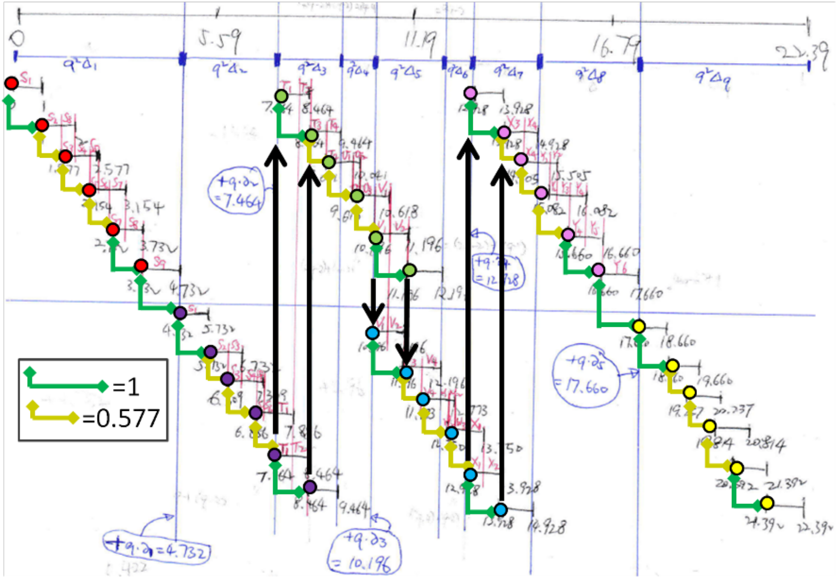

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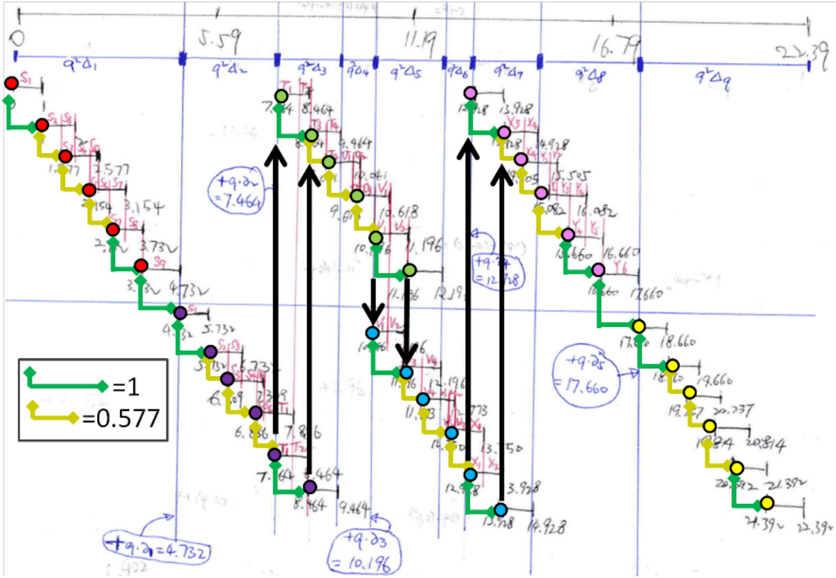

-THE END-

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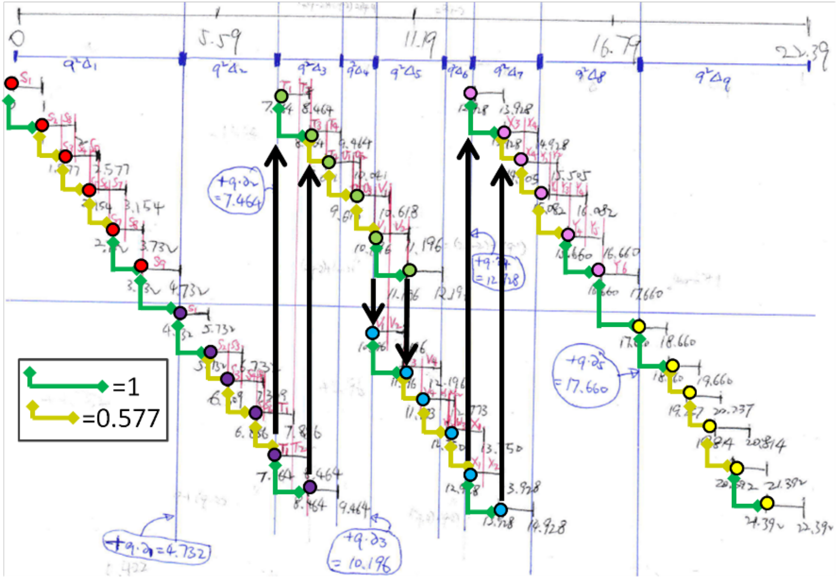

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